

OPTIMIZATION OF ATM LEASED LINE NETWORKS FROM USER'S VIEWPOINT

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The notable feature of the ATM leased line service is the freedom of virtual path bandwidth setting. Users can take advantage of this feature by optimizing their network to minimize the charge. However, such optimization is difficult because of the nonlinearity caused by statistical multiplexing. To tackle this difficulty, this paper examines the integer programming approach with piecewise linear approximation and the iterative improvement approach. It is shown that the former is valuable in evaluating the solution accuracy of the latter. The computational time and the solution accuracy of these approaches are evaluated through computer experiments.

1. INTRODUCTION

The Asynchronous Transfer Mode (ATM) technology is now sufficiently matured and is being introduced in the real world. Namely, several ATM communication services are presently available for network users. One of these services is the ATM leased line service that provides a permanent virtual path between user sites [1, 2].

For users, the essential advantage of this service is the freedom permitted in selecting the bandwidth. A user can select an arbitrary bandwidth as needed and pay for just that bandwidth. By contrast, a user must select a predefined bandwidth in Synchronous Transfer Mode (STM) services, and usually must purchase a larger bandwidth than optimum. To take full advantage of this ATM feature, a user has to optimize the bandwidth of virtual paths so as to minimize the total charge.

Some literature have treated the optimization of ATM networks [3-5]. These studies, however, treat the problem of minimizing the network operator's investment cost [4, 5] or maximizing the operator's revenue [3]. There have been no detailed studies on network optimization from the user's viewpoint.

This paper focuses on the optimization of the user's network based on the ATM leased line service. The objective function is the total charge. The offered traffic is bursty and is multiplexed statistically on a virtual path. Because of the statistical multiplexing characteristic, the problem is a difficult nonlinear non-convex optimization problem. To tackle this problem, the paper examines two approaches. One is the integer programming approach with piecewise linear approximation, while the other is the iterative improvement approach. Although neither method always yields the strict optimum, this paper shows that the integer programming approach provides the upper and lower

bounds for the exact minimum charge, and the solution accuracy is evaluated through their difference. These bounds are also useful in assessing the accuracy of the iterative improvement approach. Finally, the paper presents computational results to evaluate the accuracy and computational time of each approach.

2. PROBLEM

Suppose that a user is building his/her private network using the ATM leased line service. The user has three or more sites, each of which is equipped with an ATM router and an ATM switch. The service is provided by CBR (Continuous Bit Rate) virtual paths. For this situation, the problem is described as follows.

[Problem Definition] Which pairs of sites should be connected by virtual paths and how large should be the bandwidth of each virtual path to achieve acceptable service quality with the minimum charge (Fig. 1)?

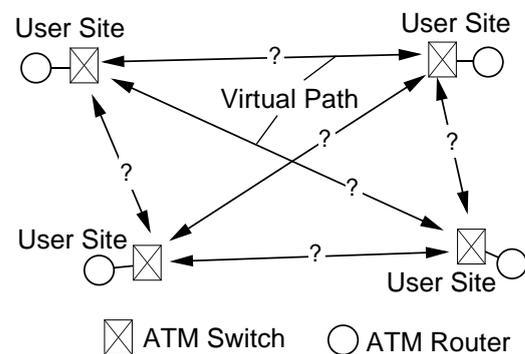


Fig. 1. Problem explanation.

- We assume the following points throughout the paper.
- An arbitrary bandwidth can be assigned to a virtual path.
 - Traffic demands are statistically multiplexed on a virtual

path.

- Traffic parameters of demands are known for all pairs of user sites.
- A virtual path can be set between any pair of user sites.
- Traffic can transit the ATM routers or switches at any user site.
- Traffic is shaped by the traffic shaper of an ATM switch at each user site as shown in Fig. 2. Thus, no cells are lost by the UPC (Usage Parameter Control) mechanism within the operator's network because the peak rate is held to the specified value at the user site.

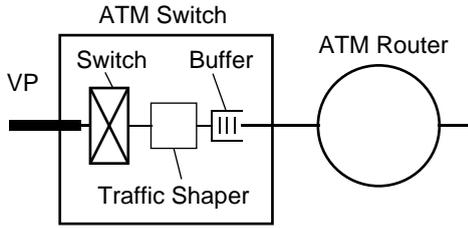


Fig. 2. Traffic shaping and buffering.

3. FORMULATION

For the problem identified in Section 2, the set of user sites is $V = \{1, 2, \dots, n\}$. Let $E = \{1, 2, \dots, m\}$ be the set of site pairs, between which a virtual path can be set up. Later, the virtual path set up between source destination pair i ($i \in E$) is simply called "virtual path i ". Denote the source and the sink of virtual path i ($i \in E$) as $a(i)$ and $b(i)$ ($a(i), b(i) \in V$), respectively. Let the set of traffic demands be $D = \{1, 2, \dots, l\}$. Also, define the source and sink of demand p ($p \in D$) as $A(p)$ and $B(p)$ ($A(p), B(p) \in V$), respectively.

To formulate the problem, let us introduce decision variable $f_{i,p}$. The value of $f_{i,p}$ is 1 if the demand p ($p \in D$) goes through the virtual path i ($i \in E$) and 0 otherwise. The bandwidth C_i required for virtual path i is a function of variables $f_{i,1}, \dots, f_{i,l}$. Since the charge for virtual path i is in turn the function of C_i , it is expressed as $Z_i(C_i(f_{i,1}, \dots, f_{i,l}))$. Using these notations, the problem is formulated as follows.

$$P: \text{minimize } \sum_{i \in E} Z_i(C_i(f_{i,1}, \dots, f_{i,l})), \quad (1)$$

subject to

$$\sum_{i \in \{i | a(i)=s\}} f_{i,p} - \sum_{i \in \{i | b(i)=s\}} f_{i,p} = \begin{cases} 1, & \text{if } s = A(p) \\ -1, & \text{if } s = B(p) \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

for $s \in V, p \in D$.

Here, eq. (2) is the flow conservation equation [6]. Obviously, this is a multi-commodity least-cost flow problem. In eq. (1), C_i and Z are nonlinear functions of $f_{i,p}$. Moreover, these functions are not convex because of the characteristic of statistical multiplexing. Reference [7] described a method for solving a multi-commodity minimum cost flow problem with such an objective function. However, this method yields only a local minimum solution and does not guarantee that the obtained solution is close to the exact solution. Thus, it is necessary to establish a new method such that the obtained solution is assured to be close to the strict minimum.

3.1. Equivalent Capacity

As assumed in Section 2, user traffic is statistically multiplexed onto virtual paths and shaped by traffic shapers. This means that cells are queued at the buffer of a traffic shaper. If the buffer overflows, cells may be lost and thus the quality of service may degrade to an unsatisfactory level. To avoid such quality degradation, the virtual path bandwidth must be appropriately determined from the traffic parameters and the quality of service criteria using the equivalent capacity (or effective bandwidth) concept [8-10].

Several equivalent capacity formulas have been proposed for different mathematical models of traffic sources. Among these formulations, this study employs the method of Ref. [8], proposed for the two state Markov source and the buffer overflow probability criteria. The advantages of this formula are:

- The method is applicable to a wide range of traffic models [8].
- The formula is relatively simple and thus easy to be treated by the piecewise linear approach.

From the equivalent capacity formula, the bandwidth of a virtual path i is expressed using variables $f_{i,p}$ as follows.

$$C_i = \min \left\{ \sum_{p \in D} \hat{c}_p f_{i,p}, \sum_{p \in D} m_p f_{i,p} + \alpha \sqrt{\sum_{p \in D} \sigma_p^2 f_{i,p}} \right\}, \quad (3)$$

where m_p is the mean rate, σ_p is the standard deviation of the rate, and \hat{c}_p is the equivalent capacity of the Fluid-Flow approximation of the traffic demand p .

In eq. (3), the first argument of the minimum function is referred to as the "Fluid-Flow" equivalent capacity, while the second argument is referred to as the "Stationary" equivalent capacity. Value \hat{c}_p is derived from parameters such as the peak rate, the mean burst

period, and the utilization of the source, the buffer size and the buffer overflow probability. The constant α is also computed from the buffer overflow probability.

3.2. Charge

This study assumes a simple charge model. First, the charge of virtual path i is assumed to be proportional to the distance from $a(i)$ to $b(i)$. Additionally, the charge consists of a part that is independent of the bandwidth and a part that is proportional to the bandwidth. The former is zero if the virtual path is not set up. Thus, the charge model of virtual path i , Z_i , is written by the next equation.

$$Z_i = \begin{cases} d_i(C_i + X), & \text{if virtual path exists} \\ 0, & \text{otherwise} \end{cases}, \quad (4)$$

where C_i is the bandwidth defined in eq.(3), and X is a constant.

4. PIECEWISE LINEAR INTEGER PROGRAMMING

4.1. Bounds for the Strict Optimum

To solve the problem, this study first employs the integer programming approach with the piecewise linear approximation. This approach is advantageous since it yields the exact solution for the approximated problem. Moreover, since integer programming technology has progressed greatly in recent years, it is now possible to take use high performance software [11]. The most important point is that the approach provides the upper and lower bounds for the objective function value of the exact solution. Therefore, it becomes possible to know whether the approximated solution is sufficiently close to the exact solution or not by comparing it to the upper and lower bounds.

Before deriving the bounds, let us introduce some additional notations. Suppose that the nonlinear function $Z_i(C_i(f_{i,1}, \dots, f_{i,l}))$ is approximated by a piecewise linear function $Z_i^*(C_i(f_{i,1}, \dots, f_{i,l}))$. Let P^* be the problem generated from P by replacing $Z_i(C_i(f_{i,1}, \dots, f_{i,l}))$ with $Z_i^*(C_i(f_{i,1}, \dots, f_{i,l}))$. Let z_{opt} and \hat{z}_{opt} be the objective function values of the exact solutions for P and P^* .

If it is possible to select the piecewise linear function such that

$$Z_i(C_i(f_{i,1}, \dots, f_{i,l})) \geq Z_i^*(C_i(f_{i,1}, \dots, f_{i,l})), \quad (5)$$

for any virtual path i and any values of $f_{i,1}, \dots, f_{i,l}$, the following theorems are derived.

[Theorem 1]

If eq. (6) holds, then

$$z_{opt} \geq \hat{z}_{opt}, \quad (6)$$

(Proof) Let $f_{i,p}^o$ be the value of $f_{i,p}$ that yields the strict optimum of P . The set of $f_{i,p}^o$ is a feasible solution for P^* , because P and P^* have the same constraint conditions. From eq.(5), the relation

$$\sum_{i \in E} Z_i(C_i(f_{i,1}^o, \dots, f_{i,l}^o)) \geq \sum_{i \in E} Z_i^*(C_i(f_{i,1}^o, \dots, f_{i,l}^o)) \quad (7)$$

holds, and its left hand is z_{opt} . Meanwhile, the right hand is the objective function value of a feasible solution for P^* and not less than \hat{z}_{opt} . This concludes eq. (6).

[Theorem 2]

Let $f_{i,p}^*$ be the value of $f_{i,p}$ that yields the strict optimum of P^* . Let z^* denote the value obtained by substituting $f_{i,p}^*$ for $f_{i,p}$ in the objective function of P . Then, if eq.(5) holds,

$$z^* \geq z_{opt}. \quad (8)$$

(Proof) The set of $f_{i,p}^*$ is a feasible solution for P . The objective function value of this feasible solution is z^* and is not less than the strict optimum. This proves eq. (8).

The upper bound z^* is easily computed from $f_{i,p}^*$ obtained by solving P^* . Thus, the solution accuracy is immediately known from z^* and \hat{z}_{opt} if P^* is solved.

4.2. Formulation to Integer Programming Problem

Two nonlinear functions in eq. (3) must be addressed for the conversion from P to P^* . These functions are the minimum function and the square root function.

To express the minimum function with linear equations, this study employs a ‘‘parallel path’’ approach shown in Fig. 3(a). Suppose that there can be one of two virtual paths between a pair of sites. The bandwidth of one virtual path $VP_{i,0}$ is specified by the Fluid-Flow equivalent capacity while that of the other path $VP_{i,1}$ is specified by the Stationary equivalent capacity. Then, consider the problem of minimizing the total path charge of the network where at most one of these two paths can be set up between two sites. For this problem, if the objective function is correctly minimized, the virtual path with less bandwidth is selected from the parallel paths. Thus, the original configuration is completely equivalent to the parallel path configuration. In the parallel path configuration, the bandwidths do not include the minimum function any more.

The square root included in the bandwidth of $VP_{i,1}$ is approximated by lines between two adjacent points on the curve $y = \sqrt{x}$ as shown in Fig. 4. In Fig. 4, the

points are labeled as (v_k, w_k) ($1 \leq k \leq K$) and the line between (v_{k-1}, w_{k-1}) and (v_k, w_k) is labeled as l_k . The approximated piecewise linear function is not larger than the square root function between (v_0, w_0) and (v_K, w_K) . Therefore, the piecewise linear approximation satisfies eq. (5), and thus it is possible to evaluate the upper and lower bounds derived by Theorems 1 and 2.

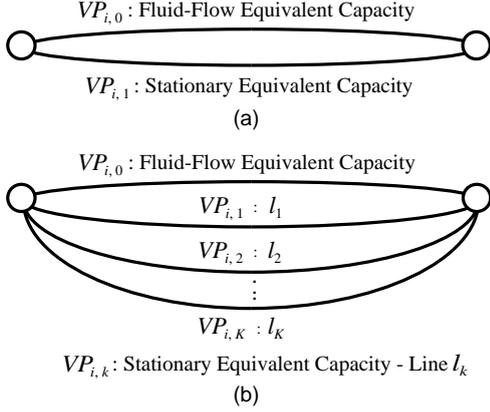


Fig. 3. Parallel path approach for the integer programming formulation.

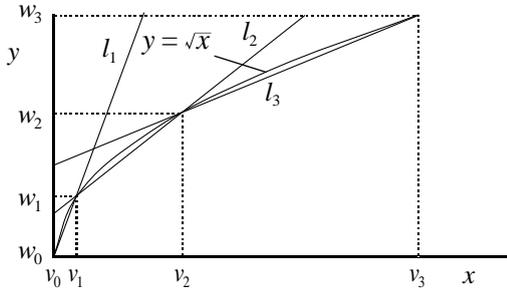


Fig. 4. Piecewise linear approximation of the square root function.

As shown in Fig. 4, line l_k is not larger than any other line l_j ($k \neq j$) in the interval $[v_{k-1}, v_k]$. This suggests the parallel path scheme is again applied. Namely, $VP_{i,1}$ in Fig. 3 (a) is further divided into a set of parallel paths, each of which has a bandwidth associated with a line l_k ($1 \leq k \leq K$) as shown in Fig. 3 (b). The path associated with line l_k ($1 \leq k \leq K$) is labeled as $VP_{i,k}$ while the path associated with the Fluid-Flow equivalent capacity is labeled as $VP_{i,0}$. Let line l_k be expressed as,

$$y = a_k x + b_k \quad (9)$$

Then, from eqs. (3) and (9), the k -th path associated with line l_k has the bandwidth $C_{i,k}$ defined by,

$$C_{i,k} = \sum_{p \in D} (m_p + \alpha a_k \sigma_p^2) f_{i,p} + b_k y_{i,k}, \quad 1 \leq k \leq K \quad (10)$$

The value of 0-1 variable $y_{i,k}$ is 1 if $VP_{i,k}$ exists and 0 otherwise. Then, if constraints are added to select at most one path among these K parallel paths, and if the objective function is successfully minimized, then the virtual path charge is exactly the same as that of the piecewise linear function shown in Fig. 4.

By paralleling a virtual path in this way, the bandwidth is now expressed by linear equations. The cost function shown in eq. (4) can be also expressed by linear equations with the addition of some integer variables. Thus, the problem is formulated into a linear integer programming problem, which can be solved by generic optimization software. For the formulation, let us use variable $f_{i,p,k}$ ($i \in E, p \in D, 0 \leq k \leq K$) that takes value 1 if demand p goes through path $VP_{i,k}$ and value 0 otherwise. Then the problem is formulated as:

$$\text{minimize } \sum_{i \in E} d_i \left(\sum_{p \in D, 0 \leq k \leq K} s_{p,k} f_{i,p,k} + \sum_{0 \leq k \leq K} t_k y_{i,k} \right), \quad (11)$$

subject to

$$\sum_{y=0}^K y_{i,k} \leq 1, \quad \text{for } i \in E, \quad (12)$$

$$f_{i,p,k} \leq y_{i,k}, \quad \text{for } i \in E, p \in D, 0 \leq k \leq K, \quad (13)$$

$$\sum_{\substack{i \in \{i|a(i)=s\} \\ 0 \leq k \leq K}} f_{i,p,k} - \sum_{\substack{i \in \{i|b(i)=s\} \\ 0 \leq k \leq K}} f_{i,p,k} = \begin{cases} 1, & \text{if } s = A(p) \\ -1, & \text{if } s = B(p), \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

for $s \in V, p \in D,$

where $s_{p,k}$ and t_k are constants defined as follows.

$$s_{p,k} = \begin{cases} m_p + \alpha a_k \sigma_p^2, & 1 \leq k \leq K \\ \hat{c}_p, & k = 0 \end{cases} \quad (15)$$

$$t_k = \begin{cases} X + \alpha b_k, & 1 \leq k \leq K \\ X, & k = 0 \end{cases} \quad (16)$$

5. ITERATIVE IMPROVEMENT

Although the integer programming approach has advantages, its computation time may not be practical for a large size problem. For such a problem, it is necessary to establish a fast heuristic optimization method. As such a method, this study investigates the following iterative improvement method.

(Definition)

N : the current solution network.

S : the set of virtual paths included in N .

T : the set of virtual paths not included in N .

Step 1) Make an initial solution using a minimum spanning tree [6] based on the physical distances among sites.

Step 2) Repeat A and B until the solution shows no further improvement.

A. Exchange step

For every combination of i ($i \in S$) and j ($j \in T$), construct network N_{ij} by deleting i from N and adding j to N . If N_{ij} is connected, assign traffic demands along the shortest routes. Next, compute the virtual path bandwidths and the network charge. Select N_{ij} with the minimum charge, N_{\min} . If the charge of N_{\min} is less than that of N , replace N with N_{\min} .

B. Addition step

For every j ($j \in T$), construct network N_j by adding j to N . Assign traffic demands along the shortest routes. Next, compute the virtual path bandwidths and the network charge. Select N_j with the minimum charge, N_{\min} . If the charge of N_{\min} is less than that of N , replace N with N_{\min} .

6. COMPUTATIONAL RESULTS

The solution accuracy and computational time of the described methods were evaluated through computer simulation. Figure 5 shows the computational time against the problem size for both methods. In the figure, the computational time is the average for five randomly generated problems for each number of sites. A problem was generated as follows. Each site was chosen randomly from the cities of Japan, and the distances between sites were those between the cities. For each pair of two sites, data bursts are generated by a two-state Markov source. Each source generates bursts according to the parameters shown in Table 1. Because the utilization parameter was determined randomly for each source, the mean rate between two sites was also random. The table shows the buffer size of a switch and the buffer overflow probability as well. In the charge of eq. (4), the constant X was 10, while the unit of bandwidth was Mbits/s. For the integer programming approach, the number of piecewise linear intervals K was 3. Figure 5 shows that the computational time is much less and increases much slower against the problem size for the iterative improvement approach. Nevertheless, the figure also implies that the integer programming approach is applicable to small problems.

Next, solution accuracy was tested for 13 site problems. With the integer programming approach, the

differences between the upper and lower bounds were 5% to 8% of the lower bounds for 10 randomly generated problems and $K=3$. Figure 6 shows that the distribution of the relative difference Δ between the upper bounds obtained by the integer programming approach and the solution obtained by the iterative improvement approach. The figure also displays how the initial solutions of the iterative improvement approach are distributed. As shown in the figure, the solutions are as good as those offered by the integer programming approach and thus close to the exact optimum. However, the initial solutions of the iterative improvement approach are not as good as the final solutions. This result suggests that the minimum spanning tree is not a very good heuristic and an improvement step is indispensable to obtain a good solution.

Table 1. Parameter values for the traffic sources and the switch buffers.

| | |
|-----------------------------|------------------------------------|
| Burst Peak Rate | 10 Mbits/s |
| Average Burst Period | 30 msec |
| Source Utilization | Uniform Distribution in $[0, 0.1]$ |
| Buffer Size | 1 Mbits |
| Buffer Overflow Probability | 10^{-5} |

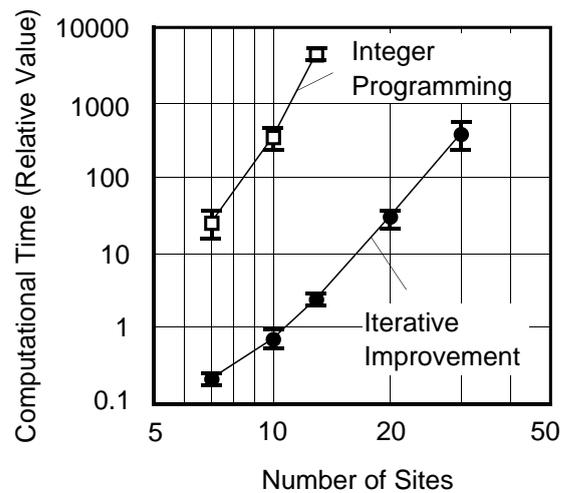


Fig. 5 Computational time.

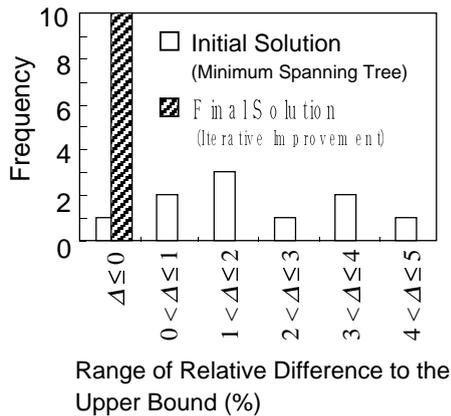


Fig. 6 The distribution of the difference between the approximate solution by the iterative improvement approach and the upper bound of the exact solution.

7. CONCLUSION

This paper described the problem of optimizing the user's network to minimize the charge for the ATM leased line service. The problem was formulated using the equivalent capacity of Ref. [8] to construct a network with sufficiently good service quality. As optimizing methods, the paper investigated the piecewise linear integer programming approach and the iterative improvement approach. The integer programming approach was shown to be advantageous in the point that the upper and lower bounds of the exact solution are easily derived and thus the solution accuracy is effectively evaluated. Computation results show that the iterative improvement approach is much faster and yields solutions that are as good as those of the integer programming approach. Thus, the iterative improvement approach is a practical choice for the optimization problem of the user ATM network. Nevertheless, the integer programming approach is still significant in confirming the solution accuracy of the iterative improvement approach or other heuristic approaches.

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